One-Dimensional Numerical Model for a Spray Column Heat Exchanger

Steven M. Summers and Clayton T. Crowe
Dept. of Mechanical and Materials Engineering, Washington State University, Pullman, WA 99164

A numerical model describing performance of a liquid-liquid spray column heat exchanger is developed. The model is for a disperse packed column operating in steady state. The fundamental equations for the continuous and distributed phases are developed in Eulerian and Lagrangian forms, respectively. The model predicts simultaneously the volume fraction, velocity, and temperature for each phase. Energy and momentum are exchanged between phases through source variables. Empiricism in the model is limited to the specification of droplet drag coefficient and Nusselt number correlations which are utilized to calculate momentum and energy source terms. Application of the computer model to column operation yields predictions that compare closely with experimental results.

Introduction

An analytical model for evaluating the performance of a countercurrent spray column heat exchanger is presented. In the model, a method of numerically calculating volume fraction or holdup at each position in the column is proposed. As a consequence of the calculation scheme used in determining the column's phase volume fraction, a convenient numerical structure is revealed for evaluating the basic equations governing momentum and heat transfer between the continuous and distributed phases.

Spray column heat exchangers have been studied for a number of applications ranging from desalination systems to petrochemical processes. In these studies, description of column performance has required evaluation of phase volume fraction using slip velocity correlations. Energy exchange is then evaluated separately from volume fraction, generally relying on specific performance correlations from various experimental columns. In the numerical model proposed, volume fraction, momentum and heat transfer calculations are performed simultaneously. This approach is advantageous because it allows investigation of how Reynolds and Nusselt numbers effect momentum and energy exchange at any position in the column.

In the model, concepts from CONVAS (conservative variable source model), originally developed by Sharma and Crowe (1978) for gas particle flows, are applied. In CONVAS, the continuous-phase governing equations are developed in Eulerian forms, while the distributed-phase governing equations are developed in Lagrangian forms. With this, complexities introduced by using a single Eulerian equation for both phases are avoided. In CONVAS, momentum and energy exchange are evaluated within a computational cell. The flow field within each cell is solved sequentially using a one-dimensional marching solution scheme. In the cell, momentum and energy exchange are evaluated at the phase interface where well-known correlations for droplet drag coefficients and droplet Nusselt numbers can be applied to determine momentum and energy transfer between phases. These correlations are used with the Lagrangian equations of the distributed phase to evaluate effects of phase interactions.

As discussed by Letan and Kehat (1970) and Kehat and Sideman (1971), spray columns can be operated in either disperse or densely packed modes, where dense and disperse refer to the way column volume fraction is controlled. In disperse packed columns, with distributed-phase volume fractions ranging from 0 to 55%, volume fraction depends solely on the momentum balance occurring between phases. In dense packed columns with distributed phase volume fractions ranging from 30 to 70% volume fraction is mechanically controlled. In dense packed columns, the advantages of the model proposed here lose significance.

To reasonably describe spray column performance, a minimum of two flow regimes in the column must be considered. The first of these is the wake growth region (Kehat and Letan,

---

Correspondence concerning this article should be addressed to C. T. Crowe.
S. M. Summers is currently with BP Exploration, Anchorage, AK.

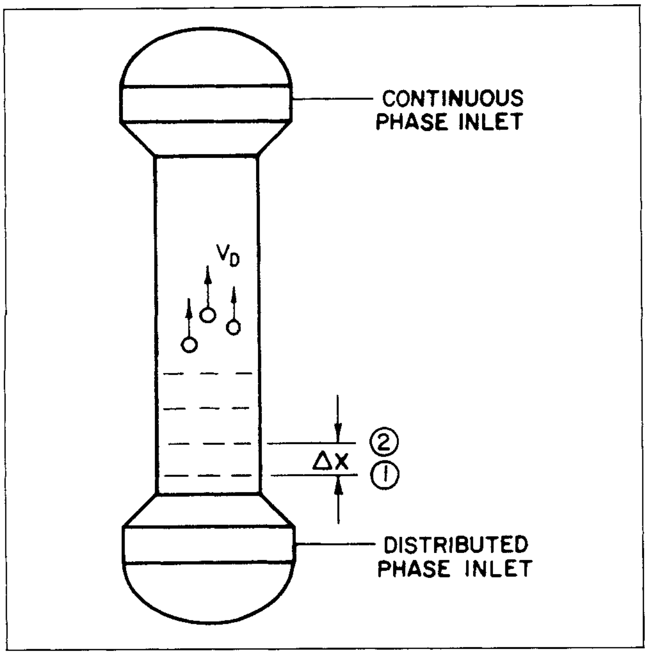

Figure 1. Typical spray configuration column with representative computational cell.

1971; Letan and Kehat, 1968a) occurring at the inlet of the distributed phase. The second region, existing throughout the remainder of the column, is that of fully developed flow where droplet wakes are shed continuously. Because of differing wake behavior in these flow regimes, two different methods for evaluating energy transfer between phases are required.

Prior to wake shedding, energy transfer occurs from the droplets to the partially developed wakes. This energy, however, is not transferred to the bulk of the continuous phase until the wake is shed (Letan and Kehat, 1965; Yeheskel and Kehat, 1971). Instead, this energy moves with the portion of continuous phase, trapped in the droplet wake, in a direction counter to the bulk of the continuous phase.

This reverse flow of continuous phase is referred to as backmixing. Backmixing becomes quite important in spray column performance because of the significant effect it has in reducing the temperature difference driving heat transfer between phases. In countercurrent kerosene and water spray columns, similar to the example developed in this article, Letan and Kehat (1965, 1968b) reported losses of phase temperature difference of 50% prior to initial wake shedding. In the numerical model, effects of wake shedding and backmixing are modeled by restricting energy transfer until conditions are identified, which would initiate wake shedding. Discussion of the methods used to accomplish this will be developed after introduction of the fundamental equations.

Fundamental Equations

The governing equations used to describe the flow regime are evaluated within the bounds of a computational cell or control volume. Consider a duct of uniform cross-section shown in Figure 1.

For the marching solution scheme proposed, the flow field is divided into similar control volumes. As the scheme proceeds, the boundary conditions of the preceding cell are redefined as initial conditions for the following cell. Phase velocity, temperature, and volume fraction are based on the solution of the momentum and energy equations within the cell calculated at the opposite cell boundary.

Eulerian continuous-phase equations

Mass Conservation Equation. The conservation equation for the continuous phase is:

$$(\rho\phi AU)_{C2} = (\rho\phi AU)_{C1} + \Delta m \tag{1}$$

where Δm is the mass transfer from the distributed phase to the continuous phase. This mass source term is set to zero for the spray column heat exchanger.

Momentum Equation. The momentum equation for the continuous phase is written as:

$$\begin{aligned}(PA)_{C2} + (\rho\phi AU^2)_{C2} \\ = (PA)_{C1} + (\rho\phi AU^2)_{C1} + \Delta M - \rho_C gA((\phi_{C1} + \phi_{C2})/2)\Delta x\end{aligned} \tag{2}$$

In this equation wall friction is neglected and the column is assumed to be of constant cross-sectional area. Where variable cross-sectional areas are considered, this equation is modified by the term $(A_2 - A_1)(P_2 - P_1)/2$. The momentum source term in this equation is ΔM, which is the momentum transferred from the distributed phase to the continuous phase.

Energy Equation. The energy equation is:

$$[(C_pT + U^2/2)UA\rho\phi]_{C2} = [(C_pT + U^2/2)UA\rho\phi]_{C1} + \Delta E \tag{3}$$

It is assumed that no heat transfer occurs to the column externally so that the only energy source to the continuous phase is from the distributed phase energy source term ΔE.

Conservative Variable. Conservative variables for the continuous phase are defined by the principle terms of the continuity, momentum and energy equations. These variables are:

$$X = (\rho\phi UA)_C \tag{4}$$

$$Y = (PA + \rho\phi AU^2)_C \tag{5}$$

$$Z = [(C_pT + U^2/2)\rho\phi AU]_C \tag{6}$$

The mass, momentum and energy equations for the continuous phase can be rewritten using source variables as:

$$X_2 = X_1 + \Delta m \tag{7}$$

$$Y_2 = Y_1 + \Delta M - \rho_C gA[(\phi_{C1} + \phi_{C2})/2]\Delta x \tag{8}$$

$$Z_2 = Z_1 + \Delta E \tag{9}$$

The equation for Y_2 assumes a linear variation in volume fraction over the distance Δx so that an average volume fraction can be used to describe the force of gravity on the continuous phase.

From the conservative variables the basic flow properties for the continuous phase can be obtained:

$$U_{C2} = U_{C1}(\phi_{C1}/\phi_{C2}) \tag{10}$$

$$P_{C2} = Y_2/A - (\rho\phi U^2)_{C2} \tag{11}$$

$$T_{C2} = [Z_2/(X_2 C_p) - U^2/(2C_p)]_{C2} \tag{12}$$

Lagrangian distributed-phase equations

The unknowns in the previous equations are the mass, momentum and energy source terms. Each of these terms can be calculated from the distributed-phase fundamental equations. Unlike the continuous-phase equations, these fundamental equations will be developed in Lagrangian forms.

Droplet Momentum Source Term. The equation for the momentum source term is:

$$\Delta M = m_{Dt}(V_{D1} - V_{D2}) \tag{13}$$

where V_{D1} and V_{D2} are distributed phase velocities at stations 1 and 2. The single unknown in this equation is V_{D2}, since V_{D1} is specified by either initial conditions or results from the previous computational cell. V_{D2} will be solved by integrating the equation of motion for the droplet.

The equation of motion for a single droplet is:

$$m_D(dV/dt) = C_d\rho\,|U_C - V_D|(U_C - V_D)A/2 + g(\rho_\phi - \rho_D)Vo_D \tag{14}$$

The modified density used in the buoyancy term is that of the fluid cloud as sensed by the droplet (Barnea and Mizrahi, 1973). This density is the effective density of the mixture and is defined as:

$$\rho_\phi = \rho_D\phi_D + \rho_C\phi_C \tag{15}$$

As a limiting case, Basset and virtual mass forces were added to the computer simulation described later. Assuming creeping flow, the influence of these terms was found to be negligible. For high Reynolds numbers, as present in the column, the influence of these forces is even less. Generally it was found that accelerations in the column are small, thus reducing the influence of these terms.

For solid particles with Reynolds numbers less than 700, a correlation for the drag on the sphere is (Yeheskel and Kehat, 1971):

$$C_d = [0.63 + 4.8/(Re)^{\frac{1}{2}}]^2 \tag{18}$$

In this equation the Reynolds number is defined as:

$$Re = \rho_C\,|U_C - V_D|\,d_D/u_C \tag{19}$$

Selection of the appropriate drag law depends on the flow conditions being considered. Certain adjustments of this correlation may be justified to account for effects such as droplet interactions. The equation of motion can be written as:

$$dV_D/dt = 18\mu_C f(U_C - V_D)/\rho_D d_D^2 + g(\rho_\phi - \rho_D)/\rho_D \tag{20}$$

where the friction factor f is defined as:

$$f = C_d(Re/24) \tag{21}$$

Defining the dynamic characteristic time as:

$$\tau_d = \rho_D d_D^2/(18\mu_C f) \tag{22}$$

the equation of motion becomes:

$$dV/dt = (U_C - V_D)/\tau_d + g(\rho_\phi - \rho_D)/\rho_D \tag{23}$$

The time derivative is now rewritten as a function of the displacement and velocity:

$$dV/dt = (dx/dt)(dV/dx) = (V)(dV/dx) \tag{24}$$

The equation can now be solved for small displacements using the improved Euler's method:

$$V_{D2}^* = V_{D1} + \Delta x[(U_{C1} - V_{D1})/(\tau_D V_{D1}) + g(\rho_\phi - \rho_D)/(\rho_D V_{D1})] \tag{25}$$

$$V_{D2} = V_{D1} + (\Delta x/2)\{[(U_{C1} - V_{D1})/V_{D1} + (U_{C1} - V_{D2}^*)/V_{D2}^*]/\tau_d + [g(\rho_\phi - \rho_D)/\rho_D](1/V_{D1} + 1/V_{D2}^*)\} \tag{26}$$

Droplet Energy Source Term. The droplet source term is:

$$\Delta E_D = m_{Dt}[C_{pD}(T_{D1} - T_{D2}) + (V_{D1}^2 - V_{D2}^2)/2] \tag{27}$$

For this equation, the only unknown is T_{D2}, which is obtained by integrating the energy equation given as:

$$m_D[d(C_p T)_D/dt] = h_t\pi d_D^2(T_C - T_D) \tag{28}$$

Rearranging it yields:

$$dT_D/dt = [(6Nu_t\kappa)_C/(d^2\rho C_p)_D](T_C - T_D) \tag{29}$$

with a thermal characteristic time defined as:

$$\tau_q = (d^2\rho C_p)_D/(6Nu_t\kappa)_C. \tag{30}$$

This equation simplifies to:

$$dT_D/dt = (T_C - T_D)/\tau_q \tag{31}$$

Rewriting the time derivative in terms of displacement results in:

$$dT_D/dx = (T_C - T_D)/(\tau_q V_D) \tag{32}$$

T_{D2} is solved for using the improved Euler method which gives:

$$T_{D2}^* = T_{D1} + \Delta x(T_{C1} - T_{D1})/(\tau_q V_{D1}) \tag{33}$$

$$T_{D2} = T_{D1} + (\Delta x/2)[(T_{C1} - T_{D1}) + (T_{C1} - T_{D2}^*)]/(\tau_q V_{D1}) \tag{34}$$

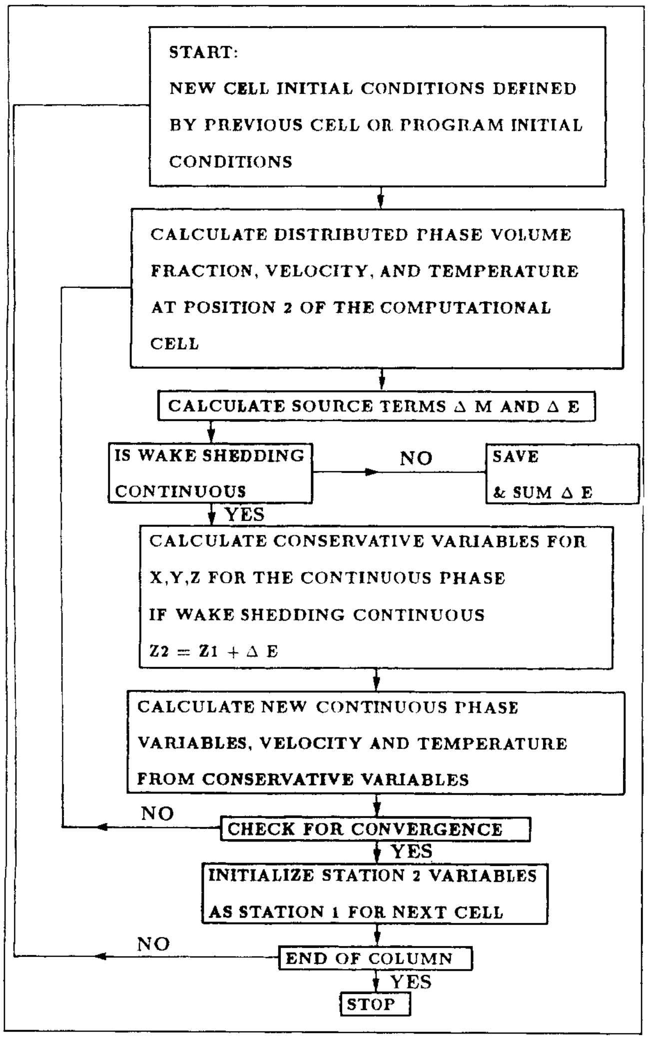

Figure 2. Computational scheme.

Two Nusselt numbers are defined for both the internal and external droplet surfaces. The external and internal Nusselt numbers are Nu_C and Nu_D and are represented (Zabel et al., 1974; Rowe et al., 1965) empirically by:

$$Nu_C = 2 + 0.60 Pr^{\frac{1}{3}} Re^{\frac{1}{2}} \quad \text{for } 30 < Re < 2{,}000 \qquad (35)$$

$$Nu_D = 0.00375[Pe_D/(1 + u_D/u_C)] \quad \text{for } Re < 1{,}000 \qquad (36)$$

The overall Nusselt number is found by summing the film coefficients in series:

$$1/h_t = 1/h_d + 1/h_c \qquad (37)$$

$$\kappa_C/Nu_t = \kappa_D/Nu_D + \kappa_C/Nu_C \qquad (38)$$

$$Nu_t = \kappa_C/(\kappa_D/Nu_D + \kappa_C/Nu_C) \qquad (39)$$

The external and internal Nusselt number correlations are for droplets that continuously shed wakes (Bird et al., 1960) and have internal circulation (Handlos and Baron, 1957). It should be noted that the external Nusselt number could be correlated to account for wake shedding or wake growth, provided experimental results were available to support adjustments.

Solution Scheme. The computational scheme is shown in Figure 2. For a spray column, the program is initiated at the column base that is assumed to be the continuous-phase outlet and the distributed-phase inlet. The distributed phase is assumed to be less dense and rise through the continuous phase. Velocity and temperature initial conditions are defined at station 1 of the first computational cell. Position 2 variables are solved for as shown in the flow chart. Station 2 variables are then redefined as station 1 variables for computations in the following sequential cell. Within each cell, energy transfer between phases is through the source variables.

Sources variables provide a means of controlling the way energy is transferred between phases. Where backmixing occurs, the energy source variable is calculated, but not transferred to the bulk of the continuous phase. Rather, it is assumed that since this energy is backmixed, the present continuous-phase temperature includes the upstream energy addition. This energy is only transferred to the continuous phase when conditions are identified that initiate wake shedding. At this point, energy previously saved in the source variable is transferred to the continuous phase. Steady wake shedding follows with energy transferred continuously between both phases in each computational cell.

Discussion and Results

The program output is modeled after experimental data by Letan and Kehat (1968a). Their data are presented in the context of the wake model, which they propose as the governing mechanism of heat transfer in the column. In the wake model, up to five regions of wake behavior are described: 1. wake growth, 2. intermediate, 3. continuous wake shedding, 4. mixing, and 5. coalescence. The model is applied to the first three regions. The latter two regions depend somewhat on column mechanical configuration and have not been modeled because they are characterized by other more specific phenomenon such as mixing and coalescence that are related only indirectly to the basic concepts presented here. In describing regions 4 and 5, the model would generally be suited to deal with the drop coalescence of region 4 along with phase mixing of region 5, provided supporting experimental correlations were available. Region 2 is described by Letan and Kehat as a region of constant distributed-phase (droplet) temperature with more or less stable wake size prior to shedding.

Computer simulations were run for the experimental conditions reported by Letan and Kehat (1968a). The predicted temperatures of the continuous (water) and distributed (kerosene) phases for the four runs reported in the article (data sets 61-2, 41-1, 68-1 and 45-2) are shown in Figures 3a through 3d. The symbol R in the figure caption is the superficial velocity ratio of the continuous to distributed phases and H is the holdup. The continuous-phase temperature is that of the bulk fluid that does not include the fluid trapped in the wake. The predicted temperatures show reasonable agreement with the measured values. No information is provided on the magnitude of the experimental error.

It should be recognized that in the simulated and experi-

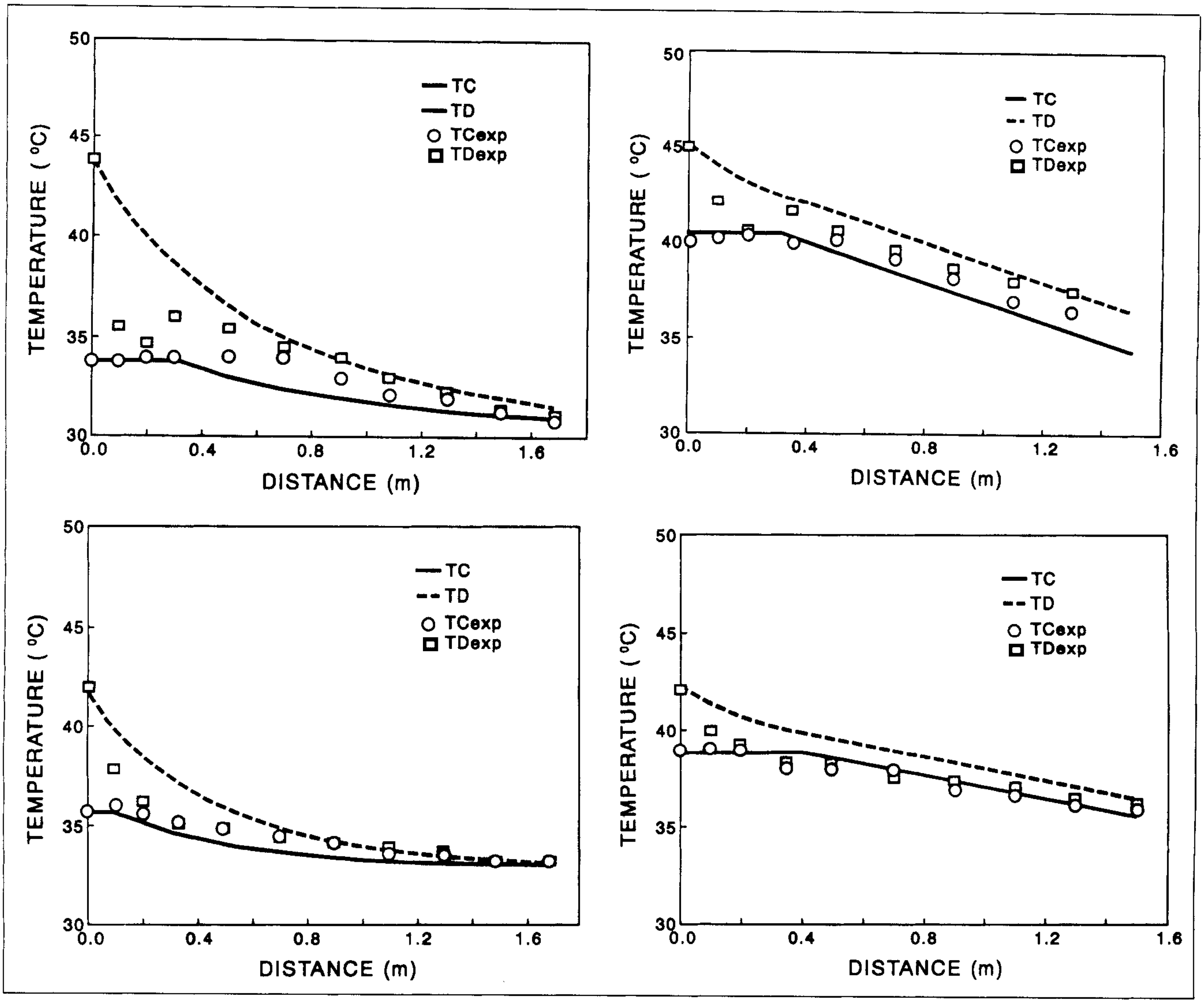

Figure 3. Predicted temperature distribution vs. experimental results for kerosene droplets in water from Letan and Kehat (1968a).

a) $R=1.0$, $H=0.06$; b) $R=2.5$, $H=0.22$; c) $R=1.0$, $H=0.43$; and d) $R=2.5$, $H=0.5$.

mental results, the distance at which the wakes are shed establishes the temperature difference, thus driving heat transfer between phases and directly affecting any comparison between simulated and experimental results. In each of the simulations, the wake shedding distance corresponded closely to experimental data with the exception of the last simulation where a wake shedding distance of 40 cm was simulated instead of 10 cm used by Letan and Kehat. At this point, it should be noted that there is some controversy as to the true nature of the different regions of wake behavior described by Letan and Kehat (Paules and Perrut, 1974). Given this controversy, discrepancies between experimental and simulated results may be justified.

Table 1. Predicted and Measured Holdup Ratios

Data Set	Superficial Velocity Ratio	Measured Holdup	Predicted Holdup
61-2	1.0	0.06	0.07
41-1	2.5	0.22	0.21
68-1	1.0	0.43	0.34
45-2	2.5	0.50	0.41

The predicted and measured holdup ratios for the four runs are shown in Table 1. The four experimental data sets have holdup ratios ranging from 0.06 to 0.5. The simulated results matched these with the exception of the last data set where the highest achievable simulated holdup was 0.42 before flooding occurred. This discrepancy may be a consequence of operating near column instability and the increased sensitivity to small differences in experimental and simulated conditions under these circumstances.

The wake model and the work of Letan and Kehat have been referenced repeatedly throughout this article. This reference is made primarily for the sake of comparing Letan and Kehat's data with the simulated results presented. Up to this point, the work here does not discuss more popular and promising theories describing column behavior such as axial mixing, backmixing, and bulk circulation. Accommodating characteristics of these theories within the model requires, to a limited extent, introduction of two-dimensional effects to the model.

The most important phenomenon may be bulk circulation, where drops concentrate in the middle of the column and rise with a velocity greater than the surrounding drops causing mixing in the continuous phase. Incorporating bulk circulation within the model could be accomplished by defining a radial velocity profile within the column and a droplet distribution associated with this profile. Momentum and energy exchange computations could then be performed, as described earlier, within subdivided cross-sectional areas defined by incremental changes in radial distance from the center of the column.

In simulating results, only one-dimensional effects have been considered. Within this context, the important parameters that define how well experimental and simulated results compare are the validity of the Nusselt number correlations, drag coefficients, and, of course, the wake shedding distances specified within the model. Adjustments of these parameters (Kumar et al., 1980; Barnea and Mizrahi, 1973, 1975; Steiner and Hartland, 1983) are only superficially related to the validity of the overall modeling approach proposed here. Regardless of the specific correlations, the fundamental physics of the modeling approach proposed remain valid.

It appears that the best utilization of this model would be in developmental studies, perhaps incorporating an experimental or industrial column. It is our belief that the model's flexibility is sufficient to incorporate all the specific parameters necessary to describe column performance. In fact, the lack of detailed phenomenological studies impose the greatest restrictions in expanding the model to more specific applications. The framework proposed is generally flexible and convenient.

Conclusion

Given the complex nature of spray column flow regimes and of multiphase systems in general, this numerical scheme provides a ready framework for spray column performance predictions. The computational scheme allows the advantages inherent in numerical models without undue complexity as may be present in multidimensional models. The model also allows direct use of the presently existing experimental data in the basic form of drag coefficients and Nusselt numbers. Considering these points, this approach presents a practical method for describing spray column performance.

Notation

A = column cross-sectional area
C_d = droplet coefficient of drag
C_p = specific heat
d = drop diameter
f = friction factor
h_c = drop outside surface heat transfer coefficient
h_d = drop inside surface heat transfer coefficient
h_t = drop total surface heat transfer coefficient
m = mass flow rate
P = pressure
U = continuous-phase velocity
V = distributed-phase velocity
V_0 = disbursed-phase drop volume
X = source variable mass exchange
Y = source variable momentum exchange
Z = source variable energy exchange

Dimensionless numbers

Re = Reynolds number
Nu = Nusselt number
Pr = Prandtl number
Pe = Peclet number

Greek letters

ΔE = energy source term
ΔM = momentum source term
Δm = mass source term
Δx = step distance
κ = thermal conductivity
μ = dynamic viscosity
ρ = density
ρ_Φ = mixture density
τ_d = dynamic time constant
τ_q = thermal time constant
ϕ = volume fraction

Subscripts

1 = station 1 variable
2 = station 2 variable
C = continuous phase
D = distributed phase
t = total
t = time

Literature Cited

Barnea, E., and J. Mizrahi, "A Generalized Approach to the Fluid Dynamics of Particulate Systems: 1," *Chem. Eng. J.*, **5**, 171 (1973).

Barnea, E., and J. Mizrahi, "A Generalized Approach to the Fluid Dynamics of Particulate Systems: 2," *Can. J. Chem. Eng.*, **53**, 461 (1975).

Bird, R., W. Stewart, and E. Lightfoot, *Transport Phenomena*, Chap. 13, p. 407, Wiley (1960).

Handlos, A., and T. Baron, "Mass and Heat Transfer from Drops Liquid-Liquid Extraction," *AIChE J.*, **3**, 127 (1957).

Kehat, E., and R. Letan, "The Role of Wakes in the Mechanism of Extraction in Spray Columns," *AIChE J.*, **17**, 984 (1971).

Kehat, E., and S. Sideman, "Heat Transfer by Direct Liquid-Liquid Contact," *Recent Advances in Liquid-Liquid Extraction*, Chap. 13, p. 455, C. Hanson, ed., Pergamon, Oxford (1971).

Kumar, A., D. Vohra, and S. Hartland, "Sedimentation of Droplets in Countercurrent Spray Column," *Can. J. Chem. Eng.*, **58**, 54 (1980).

Letan, R., and E. Kehat, "Mixing Effects in a Spray Column Heat Exchanger," *AIChE J.*, **11**, 804 (1965).

Letan, R., and E. Kehat, "The Mechanism of Heat Transfer in a Spray Column Heat Exchanger," *AIChE J.*, **14**, 398 (1968a).

Letan, R., and E. Kehat, "The Temperature Jump at the Inlet of the Continuous Phase in a Spray Column Heat Exchanger," *AIChE J.*, **14**, 831 (1968b).

Letan, R., and E. Kehat, "The Mechanism of Heat Transfer in a Spray Column Heat Exchanger: II. Dense Packing of Drops," *AIChE J.*, **16**, 955 (1970).

Paules, and M. Perrut, "Heat or Mass Transfer in Liquid Spray Columns Wake Phenomena and Entrainment Flow," *Proc. ISEC*, Lyon, France (1974).

Rowe, P., K. Claxton, and J. Lewis, "Heat and Mass Transfer from a Single Sphere in an Extensive Flowing Fluid," *Trans. Instn. Chem. Engrs.*, **43**, 14 (1965).

Sharma, M., and C. Crowe, "A Novel Physico-Computational Model for Quasi One-Dimensional Gas-Particle Flows," *J. of Fluids Eng.*, **100**, 343 (1978).

Steiner, L., and S. Hartland, "Hydrodynamics of Liquid-Liquid Spray Column," *Handbook of Fluid Motion*, Chap. 40, p. 1049, N. Cheremisinoff and R. Gupta, eds., Ann Arbor Science Publishers (1983).

Yeheskel, J., and E. Kehat, "Wakes of Vertical and Horizontal Assemblages of Drops," *Chem. Eng. Sci.*, **26**, 2037 (1971).

Zabel, T., C. Hanson, and J. Ingham, "Direct Contact Heat Transfer to Single Freely Falling Liquid Drops," *Trans. Instn. Chem. Engrs.*, **52**, 307 (1974).

Manuscript received Aug. 2, 1991, and revision received Sept. 3, 1991.